The proposed method can be used for nondestructive determination of the internal dynamic stressed state of machine components, constructions, and their elements.

In conclusion, let us note that model (3) represents a rough reflection of field nonuniformity, and fails to fully describe the real dynamic stresses within the waveguide. However, it does provide a solution for the poorly founded problem, one that is suitable for practical application. Moreover, this method of evaluating the nonuniformities of the field allows for measurement tools currently in production and, consequently, immediately available in actual practice.

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LONG WAVES OF FINITE AMPLITUDE IN POLYDISPERSED GAS SUSPENSIONS

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Most theoretical studies dealing with the wave dynamics of gas suspensions are devoted to the propagation of weak waves and waves of finite amplitude in monodisperse mixtures [1-7]. In [1, 8] we find a model for a polydisperse suspension consisting of a gas and a finite number of particle fractions. The generalization of this model to the continuous functions of particle distribution by size insofar as this relates to description of the propagation of sound waves and vapor and gas suspensions, as well as certain of the results from the calculation of dispersion and attenuation of monochromatic perturbations, is presented in [9].

It has been demonstrated in the present study that the propagation of long-wave perturbations of finite amplitude in rarefied polydisperse gas suspensions with an arbitrary mass content of particles within the mixture and an arbitrary function of particle distribution by size can be described within the framework of the model of a monodisperse medium with a particular effective particle radius. In particular, this allows us to generalize the results of the earlier analytical and numerical studies into the propagation of long waves in monodisperse suspensions without phase transitions to polydisperse gase suspensions.

1. Original Equations. Let us examine a rarefied gas suspension with a limited volumetric particle content of $\alpha_2 \ll 1$ within the mixture. The relative mass particle content $m = \alpha_2\rho_2{}^0/\rho_1{}^0$ in this case need not be small, since the true density of the material for the particle is considerably greater than the true gas density $\rho_2{}^0 \gg \rho_1{}^0$. We will assume the particles to be incompressible, and that the gas is ideal, calorically perfect (the viscosity and thermal conductivity of the gas is taken into consideration only in interphase interaction).

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The propagation of plane one-dimensional waves with the characteristic period $t_* \gg \tau$, where τ is the characteristic time required to even out the velocities and temperatures of the gas and particles, in zeroth approximation of the small parameter $\delta = \tau/t_* \ll 1$ is described by the following effective-gas model [1]:

$$d\,\rho + \rho v_x = 0, \quad \rho d_t v + p_x = 0, \quad p/p_0 = (\rho/\rho_0)^{\gamma_e},$$
$$\rho = \rho_1 + \rho_2,\ \rho v = \rho_1 v_1 + \rho_2 v_2,\ p = \rho_1 R_1 T_1$$
$$(v = v_1 = v_2,\ T = T_1 = T_2,\ d_t = \partial/\partial t + v\partial/\partial x);$$
$$c_P = \frac{c_1 + m_0 c_2}{1 + m_0}, \quad c_V = \frac{c_1 - R_1 + m_0 c_2}{1 + m_0}, \quad \gamma_e = \frac{c_P}{c_V}, \quad C_e = \left(\frac{\gamma_e p_0}{\rho_0}\right)^{1/2}$$

Here ρ, v, p, and T are the mean density, velocity, pressure, and temperature; c and R_1 is the capacity (for the gas, at constant pressure) and the gas constant; c_P and c_V are the heat capacities of the equilibrium mixture at constant pressure and volume; γ_e and C_e are the equilibrium adiabatic exponent and speed of sound in the mixture. The subscript 0 identifies the initial unperturbed state, which is assumed to be uniform through the space. The subscripts 1 and 2 denote the corresponding parameters of the gas and the particles, while quantities which do not have these subscripts characterize the mixture as a whole. The subscripts t and x denote partial derivatives with respect to the time t and the spatial coordinate x; d_t is the total (baricentric) derivative.

In the following (first) approximation of the small parameter δ we must take into consideration the velocity and thermal nonequilibrium of the phases. In this case, the difference between the velocities and temperatures of the gas and the particles will be small (proportional to δ), in connection with which, for force and thermal interphase interactions linear quasisteady laws will be valid.

Let the particles in the mixture be thoroughly mixed and distributed by size over the segment $\Delta = [a_-, a_+]$ with density $N(a, x, t)$ [in the initial state $N_0(a)$]. The velocity and temperatures of the particles will be functions of their dimensions [9]. If we assume that the particles are spherical and do not interact directly with each other, we can write a closed system of equations for the conservation of mass, momentum, and energy, as well as the equations of state, in the following form [1, 9]:

$$\begin{gathered} d_1\rho_1 + \rho_1 v_{1x} = 0,\ \tilde{d}_2\tilde{N} + \tilde{N}\tilde{v}_{2x} = 0, \\ \rho_1 d_1 v_1 = -p_x - F, \quad \tilde{d}_2\tilde{v}_2 = (v_1 - \tilde{v}_2)\,\tilde{\tau}_v^{-1}, \quad \rho_1 c_1 d_1 T_1 = \\ = d_1 p - Q + A, \quad \tilde{d}_2\tilde{T}_2 = (T_1 - \tilde{T}_2)\,\tilde{\tau}_T^{-1}; \\ F = \int\limits_\Delta \tilde{m}_2\tilde{N}(v_1 - \tilde{v}_2)\,\tilde{\tau}_v^{-1}\,da, \quad Q = \int\limits_\Delta \tilde{m}_2 c_2\tilde{N}(T_1 - \tilde{T}_2)\,\tilde{\tau}_T^{-1}\,da, \\ A = \int\limits_\Delta \tilde{m}_2\tilde{N}(v_1 - \tilde{v}_2)^2\tilde{\tau}_v^{-1}\,da, \quad p = \rho_1 R_1 T_1, \quad \tilde{m}_2 = \frac{4}{3}\pi a^3\rho_2^0 = \text{const}, \\ \tilde{\tau}_v = \frac{2}{9}\frac{\rho_2^0 a^2}{\mu_1}, \quad \tilde{\tau}_T = \frac{\rho_2^0 c_2 a^2}{3\lambda_1}\left(1 + \frac{\lambda_1}{5\lambda_2}\right); \quad d_j = \frac{\partial}{\partial t} + v_j\frac{\partial}{\partial x}, \quad j = 1, 2. \end{gathered} \tag{1.1}$$

Here F and Q are the total flows of momentum and heat from the gas to the particle; A is the work of the interphase forces; τ_V and τ_T are the quasisteady times of velocity and thermal relaxation in particles of radius a;† μ and λ are the dynamic viscosity and thermal conductivity; d_j denotes the complete derivatives along the trajectory v_j (j = 1, 2). The quantities dependent on a are identified with the symbol ~.

For purposes of subsequent analysis let us introduce the average density and the mean mass velocity of the mixture as a whole and the diffusion phase velocities

$$\rho = \rho_1 + \int\limits_\Delta \tilde{m}_2\tilde{N}da,\ \rho v = \rho_1 v_1 + \int\limits_\Delta \tilde{m}_2\tilde{N}\tilde{v}_2\,da, \tag{1.2}$$

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† τ_V is the Stokes relaxation time; τ_T is found from the three-temperature model of thermal interaction [6, 7] with dimensionless flows of heat from the particle surfaces to the gas, i.e., $Nu_1 = 2$ and from the particle surfaces to the particle phase $Nu_2 = 10$, where the subsequent effect can usually be neglected, owing to $\lambda_1 \ll \lambda_2$ (the two-temperature scheme) [7].

$$w_1 = v_1 - v,\ \tilde{w}_2 = \tilde{v}_2 - v,\ \rho_2 = \rho - \rho_1.$$

It follows from this determination that

$$\begin{gathered} \rho_1 w_1 + \int\limits_{\Delta} \tilde{m}_2 \tilde{N} \tilde{w}_2\, da = 0; \quad d_j = d_t + w_j \partial/\partial x, \quad j = 1, 2; \\ d_t\rho + \rho v_x = 0, \quad \rho d_t v = -p_x - \left(\rho_1 w_1^2 + \int\limits_{\Delta} \tilde{m}_2 \tilde{N} \tilde{w}_2^2\, da\right)_x, \end{gathered} \tag{1.3}$$

where the equations for the conservation of mass and momentum for the mixture are derived by integration of the corresponding Eqs. (1.1) over the particle masses and through addition to the conservation equations for the gas phase.

2. First Approximation of the Small Parameter δ. The equations for the conservation of momentum and energy in particles of radius a from (1.1) are ordinary differential equations, provided that we treat them along the trajectories of motion for individual particles. With small $\delta \sim \tilde{\tau}_V/t_* \sim \tilde{\tau}_T/t_*$ the solutions of these equations for $\tilde{v}_2$ and $\tilde{T}_2$ can be presented in the form of normal series

$$\begin{gathered} \tilde{v}_2 = v_1 - \tilde{\tau}_v d_2 v_1 + \ldots, \\ \tilde{T}_2 = T_1 - \tilde{\tau}_T \tilde{d}_2 T_1 + \tilde{\tau}_T{}^2 \tilde{d}_2{}^2 T_1 - \ldots \end{gathered} \tag{2.1}$$

From the first of the expressions in (2.1) and from definition (1.2) we find

$$w_1 - \tilde{w}_2 = \tilde{\tau}_v (d_t v + \tilde{d}_2 w_1 + \tilde{w}_2 v_x) + O(v\delta^2). \tag{2.2}$$

Integrating (2.2) over the particle masses and taking into consideration (1.2) and (1.3), we find that

$$w_1 = \rho^{-1} (d_t v) \int\limits_{\Delta} \tilde{m}_2 \tilde{\tau}_v \tilde{N}\, da + O(v\delta^2), \quad \tilde{w}_2 = w_1 - \tilde{\tau}_v d_t v + O(v\delta^2). \tag{2.3}$$

From the equations of the conservation of mass and number of particles it follows that

$$\rho^{-1} d_t \rho = \rho_1^{-1} d_t \rho_1 + O(\delta v_x) = \tilde{N}^{-1} d_t \tilde{N} + O(\delta v_x) = -v_x,$$

or $\tilde{N} = \rho\rho_0{}^{-1}\tilde{N}_0(1 + O(\delta))$, $m = m_0(1 + O(\delta))$ $(m = \rho_2/\rho_1)$. The general system of equations, when we utilize expressions (2.1), (2.3), (1.2), and (1.3) in first approximation of δ, can be represented in the following form:

$$\begin{gathered} d_t\rho + \rho v_x = 0,\ \rho d_t v + p_x = 0, \\ p = \rho_1 R_1 T_1,\ d_t\rho_1 + \rho_1 v_x + (\rho_1 w_1)_x = 0, \\ (\rho_1 c_1 + \rho_2 c_2) d_t T_1 - d_t p - c_2\theta_1 + w_1\rho_1(c_1 - c_2) T_{1x} = 0, \\ w_1 = m_0(1 + m_0)^{-1} \tau_v d_t v,\ \theta_1 = \rho m_0 (1 + m_0)^{-1} \tau_T d_t{}^2 T_1. \end{gathered} \tag{2.4}$$

Important is the circumstance that we have eliminated the need for the equations of conservation of the number of particles with radius a (the equation for the distribution density $\tilde{N}$), since only the integral characteristic $N_0(a)$ of the initial distribution within the average relaxation times τ_V and τ_T enter into the remaining system:

$$\begin{gathered} \tau_j = \left(\int\limits_{\Delta} \tilde{m}_2 \tilde{\tau}_j \tilde{N}_0\, da\right) \Big/ \left(\int\limits_{\Delta} \tilde{m}_2 \tilde{N}_0\, da\right), \quad j = v, T, \\ \tau_v = \frac{2}{9} \rho_2^0 a_{5,3}^2 \mu_1^{-1}, \quad \tau_T = \frac{1}{3} \rho_2^0 c_2 a_{5,3}^2 \lambda_1^{-1} \left(1 + \frac{1}{5} \lambda_1 \lambda_2^{-1}\right). \end{gathered} \tag{2.5}$$

Here $a_{5,3}$ is the mean radius from the series [9]/

$$a_{m,n} = \left[\left(\int\limits_{\Delta} N_0(a)\, a^m\, da\right) \Big/ \left(\int\limits_{\Delta} N_0(a)\, a^n\, da\right)\right]^{1/(m-n)}. \tag{2.6}$$

3. Certain Simplifications. Up until now, when we expand over the small parameter δ, we imposed no limitations on the amplitudes of the waves being dealt with here. System (2.4) can be significantly simplified if we assume that the relative amplitudes of the perturbations are finite, i.e., $\varepsilon \ll \delta^{-1}$, where $\varepsilon \sim (p - p_0)/p_0 \sim (\rho - \rho_0)/\rho_0 \sim (T - T_0)/T_0$. In this case, we can neglect the last terms in the equation for the conservation of energy as small in comparison with the rest, while in the terms proportional to δ we can take the unperturbed parameters of the medium as coefficients. In particular, the operator d_t in w_1 and θ_1 from (2.4) is commutative with respect to $\partial/\partial x$: $d_t(\partial/\partial x) = (\partial/\partial x)d_t$.

In the place of the variable ρ_1 let us introduce the relative mass content of the particles in the mixture, namely $m = \rho_2/\rho_1$, $\rho = \rho_1(1 + m)$. Then, from the continuity equation (2.4), with consideration of (2.5) and (1.2), we have

$$d_t m = m_0 \tau_v d_t v_x,\ m = m_0(1 + \tau_v v_x),$$

i.e., the perturbation m is a quantity of the first order of smallness with respect to δ. Making use of this fact, we can bring the equation for the conservation of energy to the form

$$\begin{gathered} d_t p = -\gamma_e p v_x - \rho_{10} m_0 C_e{}^2 \psi_0 \tau_v d_t v_x, \\ \psi_0 = 1 + c_2\left(\frac{1}{c_V} - \frac{1}{c_P}\right)\frac{\tau_T}{\tau_v} = 1 + \frac{\frac{3}{2}(\gamma_1 - 1)\bar{c}^2 \operatorname{Pr}_1\left(1 + \frac{1}{5}\bar{\lambda}\right)(1 + m_0)}{(1 + m_0\bar{c})(1 + \gamma_1 m_0 \bar{c})}, \\ \bar{c} = c_2/c_1,\ \bar{\lambda} = \lambda_1/\lambda_2,\ \operatorname{Pr}_1 = \mu_1 c_1/\lambda_1,\ \gamma_1 = c_1/(c_1 - R_1). \end{gathered} \tag{3.1}$$

When we substitute the v_x from the continuity equation into (3.1), we see that the relationship can be singly integrated over t and we can derive the equation of state $p = p(\rho, d_t\rho)$. We can thus present system (2.4) in the following canonical form:

$$\begin{gathered} d_t\rho + \rho v_x = 0,\ \rho d_t v + p_x = 0, \\ p/p_0 = (\rho/\rho_0)^{\gamma_e} + \gamma_e \tau_0 d_t \rho/\rho_0. \end{gathered} \tag{3.2}$$

Here we have introduced the characteristic relaxation time for the average density of the mixture, i.e., $\tau_0 = \tau_V \psi_0 m_0/(1 + m_0)$.

In the Lagrange variables t and ξ, where $v = d_t x$ and the operator d_t becomes the operator of partial differentiation, Eq. (3.2) reduces to a single equation for the dimensionless density, namely $\bar{\rho} = \rho/\rho_0$:

$$(\bar{\rho}^{-1})_{tt} + \gamma_e^{-1} C_e^2 (\bar{\rho}^{\gamma_e})_{\xi\xi} + C_e^2 \tau_0 \bar{\rho}_{t\xi\xi} = 0. \tag{3.3}$$

For small perturbations $\bar{\rho} = 1 + \varepsilon R$, $\varepsilon \ll 1$, from (3.3) we have

$$R_{tt} - C_e{}^2 R_{\xi\xi} - C_e{}^2 \tau_0 R_{t\xi\xi} = 0. \tag{3.4}$$

Linearization of (3.2) leads to (3.4), where R may be understood to refer to perturbations in density, pressure, or velocity, while differentiation with respect to ξ can be identified with differentiation with respect to x. The dispersion relationship which corresponds to Eq. (3.4) is in agreement with the low-frequency asymptote of the complex wave number for polydisperse suspensions [9] (in [9] ψ_0 does not contain $\bar{\lambda}$ owing to the smallness of this quantity).

If we regard the relaxation term in the equation of state (3.2) as a small correction factor perturbing the Riemann wave, system (3.2) in the $\varepsilon\delta \ll 1$ approximation can be reduced to the Burger equation for velocity [10], and in the case of a wave traveling to the right in coordinates η and t, where $\eta = x - C_e t$, has the form

$$v_t + \frac{\gamma_e + 1}{2} v v_\eta = \frac{\tau_0}{2} C_e^2 v_{\eta\eta}. \tag{3.5}$$

Carrying out in (3.5) the substitution which corresponds to the Riemann wave $v = 2(\gamma_e - 1)^{-1} C_e\left[(\rho/\rho_0)^{(\gamma_e - 1)/2} - 1\right]$ and $\rho/\rho_0 = (p/p_0)^{1/\gamma_e}$, we can derive evolutionary equations for density and pressure (the dissipative terms in this case can be linearized with respect to ε).

Equation (3.5) allows us to introduce the concept of viscosity and thermal conductivity for the effective gas, since analogous equations are used to describe the propagation of weakly nonlinear waves in a gas having the following coefficients (see [10]):

$$\zeta_e + \frac{4}{3}\mu_e = \frac{\rho_0 C_e^2 m_0}{1+m_0}\tau_v, \quad \lambda_e = \frac{\rho_0 c_2 C_e^2 m_0}{1+m_0}\tau_T$$

$$\left(\zeta_e + \frac{4}{3}\mu_e + \lambda_e\left(c_V^{-1} - c_P^{-1}\right) = \rho_0 C_e^2 \tau_0\right),$$

where ζ_e and μ_e represent the volumetric and dynamic viscosities and λ_e is the thermal conductivity of the effective gas (to some extent this is a conditional breakdown).

We should note that Eq. (3.5) is in agreement with the Burger's equation for the gas velocity v_1, obtained in [5].

4. Conclusion. We have demonstrated that the propagation of long waves of finite amplitudes in a polydisperse suspension will be the same as in a monodisperse gas suspension and having identical thermophysical phase properties and a particle radius of $a_{5,3}$. This conclusion cannot be extended to short waves or to waves of moderate duration, since for the case of weak waves when $\delta \gg 1$ it is the $a_{3,2}$ radius that is effective, while in the sound dispersion region $\delta \sim 1$ the model of the monodisperse gas suspension may yield results qualitatively different from those derived within the scope of the model for the polydisperse mixture [9].

The radii $a_{m,n}$, determined from relationships (2.6), exhibit the property of symmetry $a_{m,n} = a_{n,m}$, lie on the segment Δ, and for a fixed n form an ordered sequence $a_{m,n} > a_{l,n}$ for $m > \ell$ (the latter can be proved by employing the Hölder inequality). In particular, this means that $a_{5,3} \geqslant a_{3,2} \geqslant a_{3,0} \geqslant a_{2,0} \geqslant a_{1,0}$. In other words, the radius $a_{5,3}$ for suspensions that are nonmonodisperse exceeds the mathematical expectation $a_{1,0}$, the mean-surface and mean-volume radii $a_{2,0}$, $a_{3,0}$ and the "volume-surface" radius $a_{3,2}$, which are measured, as a rule, in the experiments.

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